

DEPARTMENT OF MATHEMATICS

COURSE MODULE

FOR

MATHEMATICS (HONOURS) COURSE

New Syllabus
2016 Onwards

Part –I Paper – I

Group – A Classical Algebra (30 Marks)

Module 1

No. of Classes: 10

Inequalities: Arithmetic mean, geometric mean and harmonic mean; Schwarz inequality and Weierstrass's inequality. Simple continued fraction and its convergence, representation of real numbers.

Module 2

No. of Classes: 10

Complex numbers: De Moivre's theorem, roots of unity, exponential function, Logarithmic function, Trigonometric function, hyperbolic function and inverse circular function. Summation of Series.

Module 3

No. of Classes: 10

Polynomial: polynomial equation, Fundamental theorem of algebra (statement only), multiple roots, statement of Rolle's theorem only and its application, equation with real coefficients, complex roots, Descartes's rule of sign.

Module 4

No. of Classes: 10

Relation between roots and coefficients, transformation of equation, Reciprocal equations, special roots of unity, solution of cubic equations- Cardan's method, solution of biquadratic equation – Ferrari's method.

Group – B Abstract Algebra – I (30 Marks)

Module 5:

No. of Classes: 12

Prerequisite: Surjective, injective and bijective mapping, composition of two mappings, inverse mapping, extension and restriction of mappings, equivalence relation.

Module 6:

No. of Classes: 12

Partition of a set, countable and uncountable sets, countability of rational numbers and uncountability of real numbers. Group: Definition, examples, subgroups, necessary and sufficient condition for a nonempty set to be a subgroup, generator of a group and a subgroup, order of a group and order of an element, Abelian group. Permutation group, cycles, length of a cycle, transposition, even and odd permutation, alternating group, important examples such as S_3 and K_4 (Klein 4-group).

Module 7

No. of Classes: 12

Cyclic subgroups of a group, cyclic groups and their properties, groups of prime order, coset, Lagrange's theorem. Ring, subring, integral domain, elementary properties, field, subfields, characteristic of a field or integral domain, finite integral domain, elementary properties.

Group C Geometry of two dimensions (15 Marks)

Module 8

No. of Classes: 10

Historical aspects of Geometry. Fundamental concepts of Geometry: Euclid's postulates. Cartesian Frame of reference. Transformation of rectangular coordinate axes using matrix treatment: Translation, Rotation and both. Theory of invariants using matrix method. General second degree equation. Reduction to its normal form. Classification of conics. Pair of tangents. Chord of contacts.

Module 9

No. of Classes: 10

Pole and polar, Conjugate points and conjugate lines. Diameter and conjugate diameter. Pair of straight lines. Homogeneous second degree equation. Angle between them. Bisectors of angles of pair of lines. Condition that a second degree equation represents a pair of lines. Point of intersection. Pair of lines through the origin and the points of intersection of a line with a conic. Polar equation of a conic, tangent, normals, chord of contact. 5

Group D Geometry of three dimensions (25 Marks)

Module 10

No. of Classes: 20

Fundamental concepts. Orthogonal Cartesian Frame of reference. Coordinate system. Orthogonal projection. Direction cosines and ratios]. Transformations of rectangular coordinate axes using matrix treatment: Translation, Rotation and rigid motion. Theory of invariants using matrix method. General second degree equation involving three variables. Reduction to its normal form. Classification of surfaces. Plane. Various form of equations of planes. Pair of planes. Angle between them. Bisectors of angles of pair of lines. Condition that a second degree equation represents a pair of planes. Point of intersection. Condition of perpendicularity and parallelism of pair of planes. Straight line. Symmetric and non-symmetric form of straight line and conversion of one into another form. Angle between two straight lines. Distance of a point from a line. Angle between a line and a plane. Coplanarity of two lines. Shortest distance between two lines and its equation. Position of a line relative to a plane. Lines intersecting a number of lines. Tetrahedron. Sphere, Cone, Cylinder. Condition that a general second degree equation represents these surfaces. Section of these surfaces by a plane.

Module 11

No. of Classes: 14

Circle. Generators. Sphere through a circle. Radical plane. Tangent plane. Tangent line. Normal. Enveloping cone and cylinder. Reciprocal cone. Surfaces of revolution. Ellipsoid. Hyperboloid of one and two sheets. Elliptic Paraboloid. Hyperbolic paraboloid. Normal forms. Tangent Plane. Normal line. Generating lines and their several properties.

Summary:

Total No. of Lectures: 130

Number of Tutorials: 10

No. of Class Tests: 04

Paper – II

Group – A Analysis – I (30 marks)

Module 1

No. of Classes: 5

A brief discussion on the real number system: Field structure of \mathbb{R} , order relation, order completeness properties of \mathbb{R} . Arithmetic continuum, geometric continuum, Archimedean properties, interior points, open sets, limit points, closed sets, closure.

Module 2

No. of Classes: 10

Sequence, limit of a sequence, convergence, divergence (only definitions and simple examples). Bounded functions, monotone functions. Limit of a function at a point. Continuity of a function at a point and on an interval. Properties of continuous functions over a closed and bounded interval. Uniform continuity.

Module 3

No. of Classes: 10

Derivative of a function. Successive differentiation, Leibnitz's theorem, Rolle's theorem, mean value theorems. Intermediate value property, Darboux theorem. Taylor's theorem, and Maclaurin's theorem with Lagrange's and Cauchy's forms of remainders. Taylor's series. Expansion of elementary functions such as e^x , $\cos x$, $\sin x$, $(1+x)^n$, $\log_e(1+x)$ etc. Indeterminate forms. L'Hospital's theorem.

Module 4

No. of Classes: 10

Envelope, asymptote, curvature. Curve tracing: Astroid, cycloid, cardioids, folium of Descartes. Maxima, minima, concavity, convexity, singularity.

Module 5

No. of Classes: 10

Functions of several variables (two and three variables). Continuity and differentiability. Partial derivatives. Commutativity of the orders of partial derivatives. Schwarz's theorem, Young's theorem, Euler's theorem.

Group – B Integral Calculus (20 marks)

Module 6

No. of Classes: 10

Definite Integral – Definition of Definite Integral as the Limit of a Sum; Fundamental Theorem of Integral Calculus (statement only). General Properties of Definite Integral; Integration of Indefinite and Definite Integral by Successive Reduction.

Module 7**No. of Classes: 05**

Multiple Integral – Definition of Double Integral and Triple Integral as the Limit of a Sum; Evaluation of Double Integral and Triple Integral; Fubini's Theorem (statement and applications).

Module 8**No. of Classes: 10**

Applications of Integral Calculus – Quadrature and Rectification; Intrinsic Equations of Plane Curves; Evaluation of Lengths of Space Curves, Areas of Surfaces and Volumes of Solids of Revolution. Evaluation of Centre of Gravity of some Standard Symmetric Uniform Bodies: Rod; Rectangular Area, Rectangular Parallelepiped, Circular Arc, Circular Ring and Disc, Solid and Hollow Spheres, Right Circular Cylinder and Right Circular Cone.

Group – C , Ordinary Differential Equations (40 marks)**Module 9****No. of Classes: 10**

Picard's existence theorem (statement only) for $\frac{dy}{dx} = f(x, y)$ with $y = y_0$ at $x = x_0$. Exact differential equations, condition of integrability. Equation of first order and first degree-exact equations and those reducible to exact form. Equations of first order higher degree-equations solvable for $p = \frac{dy}{dx}$, equations solvable for y , equation solvable for x , singular solutions, Clairaut's form. Singular solution as envelope to family of general solution to the equation.

Module 10**No. of Classes: 20**

Linear differential equations of second and higher order. Two linearly independent solutions of second order linear differential equation and Wronskian, general solution of second order linear differential equation, solution of linear differential equation of second order with constant coefficients. Particular integral for second order linear differential equation with constant coefficients for polynomial, sine, cosine, exponential function and for function as combination of them or involving them. Method of variation of parameters for P.I. of linear differential equation of second order. Homogeneous linear equation of n -th order with constant coefficients. Reduction of order of linear differential equation of second order when one solution is known.

Module 11**No. of Classes: 10**

Simultaneous linear ordinary differential equation in two dependent variables. Solution of simultaneous equations of the form $dx/P = dy/Q = dz/R$. Equation of the form (Paffian form) $Pdx + Qdy + Rdz = 0$. Necessary and sufficient condition for existence of integrals of the above.

Group – D , Partial Differential Equations (10 marks)

Module 12

No. of Classes: 10

Formulation of partial differential equation, Lagrange's Linear equation. General integral and complete integral. Integral surface passing through a given curve.

Summary:

Total No. of Lectures: 130

Number of Tutorials: 10

No. of Class Tests: 04

**Part –II
Paper –III**

Group – A Abstract Algebra – II (20 Marks)

Module 1

No. of Classes: 10

Normal subgroups, properties of normal subgroups, homomorphism between the two groups, isomorphism, kernel of a homomorphism, first isomorphism theorem, isomorphism of cyclic groups. Ideal of a Ring (definition, examples and simple properties).

Module 2

No. of Classes: 5

Partial order relation, Poset, maximal and minimal elements, infimum and supremum of subsets, Lattices, definition of lattice in terms of meet and join, equivalence of two definitions.

Module 3

No. of Classes: 10

Boolean algebra, Huntington postulates, examples, principle of duality, atom, Boolean function, conjunctive normal form, disjunctive normal form, switching circuits.

Group – B Linear Algebra (30 Marks)

Module 4

No. of Classes: 15

Matrices of real and complex numbers: Prerequisite [Algebra of matrices. Symmetric and skew-symmetric matrices]. Hermitian and skew-Hermitian matrices. Orthogonal matrices. Determinants: Prerequisite [Definition, Basic properties of determinants, Minors and cofactors]. Laplace's method. Vandermonde's determinant. Symmetric and skew symmetric determinants. (No proof of theorems). Adjoint of a square matrix. Invertible matrix, Non-singular matrix. Inverse of an orthogonal Matrix. Elementary operations on matrices. Echelon matrix. Rank of a matrix. Determination of rank of a matrix (relevant results are to be state only). Normal forms. Elementary matrices. Statements and application of results on elementary matrices. Congruence of matrices (relevant results are to be state only), normal form under congruence, signature and index of a real symmetric matrix.

Module 5

No. of Classes: 20

Vector space: Definitions and examples, Subspace, Union and intersection of subspaces. Linear sum of two subspaces. Linear combination, independence and dependence. Linear span. Generators of vector space. Dimension of a vector space. Finite dimensional vector space. Examples of infinite dimensional vector spaces. Replacement Theorem, Extension theorem. Extraction of basis. Complement of a subspace.

Row space and column space of a matrix. Row rank and column rank of a matrix. Equality of row rank, column rank and rank of a matrix. Linear homogeneous system of equations : Solution space. Necessary and sufficient condition for consistency of a linear non-homogeneous system of equations. Solution of system of equations (Matrix method).

Module 6

No. of Classes: 5

Linear Transformation on Vector Spaces: Definition of Linear Transformation, Null space, range space of an Linear Transformation, Rank and Nullity, Rank-Nullity Theorem and related problems.

Module 7

No. of Classes: 5

Diagonalization: Eigen values and eigenvectors, Statement of Cayley–Hamilton theorem and its application, Diagonalization of matrices of order 2 and 3 with application to Geometry.

Group – C Number Theory (20 Marks)

Module 8

No. of Classes: 10

Well ordering principle for \mathbb{N} , Division algorithm, Principle of mathematical induction and its applications. Primes and composite numbers, Fundamental theorem of arithmetic, greatest common divisor, relatively prime numbers, Euclid's algorithm, least common multiple.

Module 9**No. of Classes: 10**

Congruences : properties and algebra of congruences, power of congruence, Fermat's congruence, Fermat's theorem, Wilson's theorem, Euler's theorem (generalization of Fermat's theorem), Linear congruence, system of linear congruence theorem. Chinese remainder theorem.

Module 10**No. of Classes: 10**

Number of divisors of a number and their sum, least number with given number of divisors. Eulers ϕ function, properties of ϕ function, arithmetic function, Mobius μ - function, relation between ϕ function and μ function. Diophantine equations of the form $ax+by = c$, a, b, c integers.

Group D Analysis-II (30 marks)**Module 11****No. of Classes: 10**

Definition of Riemann integration. Uniqueness. Darboux theory of Riemann integration. Equivalence of the two definitions. Darboux theorem (proof not required). Properties of Riemann integral. Riemann integrability of continuous function, monotone function and function having countable number of discontinuities, functions defined by the integral, their continuity and differentiability.

Module 12**No. of Classes: 10**

Fundamental theorem of integral calculus. Equivalence of Riemann integral and the anti derivative (i.e., integration as inverse process of differentiation) for continuous functions. First and second mean value theorems of integral calculus integration by parts for Riemann integrals.

Module 13**No. of Classes: 10**

Improper integral and their convergence (for unbounded functions and for unbounded range of integration) Abel's and Dirichlet's test. Beta and Gamma functions. Evaluation of improper integrals and integrals dependent on them.

Summary:**Total No. of Lectures: 130****Number of Tutorials: 10****No. of Class Tests: 04**

Paper –IV Group – A Vector Analysis (30 Marks)

Module 1

No. of Classes: 4

Prerequisites: Vector Algebra: Addition of vectors, scalar and vector products of two vectors, representation of a vector in E_3 , components and resolved parts of vectors. Point of division of a line segment, signed distance of a point from a plane, vector equation of a straight line and a plane, shortest distance between two skew lines.

Module 2

No. of Classes: 14

Product of vectors: Scalar and vector triple products, product of four vectors. Applications of vector algebra - (i) in geometrical and trigonometrical problems (ii) to find work done by a force, moment of a force about a point and about a line (iii) to calculate volume of a tetrahedron. Continuity and differentiability of vector-valued function of one variable. Velocity and acceleration. Space curve, arc length, tangent, normal. Integration of vector-valued function of one variable. Serret-Frenet Formula.

Module 3

No. of Classes: 12

Vector-valued functions of two and three variables, gradient of scalar function, gradient vector as normal to a surface. Divergence and curl, their properties. Evaluation of line integrals of the type

$$\int_C \varphi(x, y, z) d\vec{\gamma}, \int_C \vec{F} \cdot d\vec{\gamma}, \int_C \vec{F} \times d\vec{\gamma}$$

Green's theorem in the plane. Gauss and Stokes theorems (Proof not required), Green's first and second identities. Evaluation of surface integrals.

Group-B Dynamics of a Particle (Marks: 50)

Module 4

No. of Classes: 5

Prerequisite: Basic concepts of Dynamics: Motion in a straight line with uniform acceleration, Vertical motion under gravity, Momentum of a body, Newton's laws of motion, Reaction on the lift when a body is carried on a lift moving with an acceleration.

Module 5

No. of Classes: 10

Motion of two bodies connected by a string, Composition and resolution of velocities, Relative velocity and relative acceleration. Work, Power and Energy: Work, Power, Energy, Principle of energy, Conservative and non-conservative forces, Kinetic and potential energy.

Module 6

No. of Classes: 20

Principle of conservation of energy, Verification of principle of conservation of energy for a particle (i) moving along a straight line under a constant force, (ii) falling from rest under gravity, (iii) moving down a smooth inclined plane under gravity alone, (iv) projected in vacuum from the horizon with a constant velocity. Impulse and Impulsive forces: Impulse, Impulsive forces, Change of

momentum under impulsive forces, Principle of conservation of linear momentum, Motion of a shot and gun, Impulsive tension in a string, Principle of angular momentum. Collision of elastic bodies: Direct and oblique impacts, Newton's experimental law of impact, Direct and oblique impacts of a smooth sphere on a fixed horizontal plane, Direct and oblique impacts of two smooth spheres, Loss of kinetic energy due to impact, Projection of a ball from a horizontal plane.

Module 7

No. of Classes: 10

Rectilinear motion: Motion under repulsive force (i) proportional to distance (ii) inversely proportional to square of the distance, Motion under attractive force inversely proportional to square of the distance, Motion under gravitational acceleration. Simple Harmonic Motion: Simple harmonic motion, Compounding of two simple harmonic motions of the same period, Elastic string and spiral string, Hook's law, Particle attached to a horizontal elastic string, Particle attached to a vertical elastic string, Forced vibrations, Damped harmonic oscillations, Damped forced oscillations.

Module 8

No. of Classes: 10

Two dimensional motion: Angular velocity and angular acceleration, Relation between angular and linear velocity, Radial and transverse components of velocity and acceleration, Velocity and acceleration components referred to rotating axes, Tangential and normal components of velocity and acceleration, Motion of a projectile under gravity (supposed constant).

Module 9

No. of Classes: 15

Central orbits: Motion in a plane under central forces, Central orbit in polar and pedal forms, Rate of description of sectorial area, Different forms of velocity at a point in a central orbit, Apse, apse line, apsidal distance, apsidal angle, Law of force when the centre of force and the central orbit are known, Differential equation and classifications of paths under central accelerations, Stability of circular orbits, Conditions for stability of circular orbits under central force (general case). Planetary motion: Newton's law of gravitation, Kepler's laws of planetary motion, Modification of Kepler's third law, Escape velocity, Time to describe a given arc of an orbit.

Module 10

No. of Classes: 10

Motion in a resisting medium & Constrained motion: Motion of a heavy particle on a smooth curve in a vertical plane, Motion under gravity with resistance proportional to some integral power of velocity, Motion of a projectile in a resisting medium Terminal velocity, Motion of a particle in a plane under different laws of resistance, Motion on a smooth cycloid in a vertical plane, Motion of a particle along a rough curve (circle, cycloid).

Group C Tensor Calculus (20 Marks)

Module 11

No. of Classes: 10

Historical study of tensor. Concept of E^n . Tensor as a generalization of vector in E^2 , E^3 & E^n , Einstein's Summation convention. Kronecker delta. Algebra of tensor: Invariant. Contravariant and covariant vectors. Contravariant, covariant and mixed tensors. Symmetric and skew-symmetric

tensors. Addition, subtraction and scalar multiplication of tensors. Outer product, inner product and contraction. Quotient law.

Module 12

No. of Classes: 10

Calculus of tensor: Riemannian space. Line element. Metric tensor. Reciprocal metric tensor. Raising and lowering of indices. Associated tensor. Magnitude of vector. Angle between two vectors. Christoffel symbols of different kinds and laws of transformations. Covariant differentiation. Gradient, divergence, curl and Laplacian. Ricci's theorem. Riemann-Christoffel curvature tensor. Ricci tensor. Scalar curvature. Einstein's space (Definition only).

Summary:

Total No. of Lectures: 130

Number of Tutorials: 10

No. of Class Tests: 04

Part –III
Paper –V
Group – A Analysis – III (50 marks)

Module 1

No. of Classes: 15

Sequence of real numbers. Notion of convergence and limit. Monotone sequences subsequences and their convergence, upper and lower limits of a sequence, algebra of limit superior and limit inferior. Cauchy's general principle of convergence. Bolzano-Weierstrass theorem, Heine-Borel theorem.

Module 2

No. of Classes: 25

Series of non negative terms. Test for convergence: Comparison test, Ratio test, Cauchy's root test, Raabe's test, Logarithmic test, Gauss's test, Cauchy's condensation test. Alternating series, Leibnitz's test. Series of arbitrary numerical terms. Absolutely and conditionally convergent series, Riemann's rearrangement theorem (Proof not required) Sequences and series of functions and their convergence. Uniform convergence. Cauchy's criterion of uniform convergence. Continuity of a limit function of a sequence of continuous functions. Continuity of the sum function of a uniformly convergent series of continuous functions. Term-by-term differentiation and integration of a uniformly convergent series of functions.

Module 3

No. of Classes: 20

Fourier series of a function. Dirichlet's condition (statement only). Uniformly convergent trigonometric series as a Fourier series. Riemann-Lebesgue theorem on Fourier series. Series of odd and even functions. Convergence of Fourier series of piece-wise monotone functions (Proof not required) Functions of several variables (two and three variables): Theory of maxima and minima, Lagrange's method of multiplier. Jacobian, Implicit function theorem (Proof not required). Inverse function theorem (statement only). Change of variables of multiple integrals. Differentiation and integrals under the sign of integration. Integral as a function of parameter. Change of order of integration for repeated integrals.

Group-B Complex Analysis (20 Marks)

Module 4

No. of Classes: 15

Introduction of complex numbers as ordered pair of real numbers (a, b) and their representation as $a + ib$, the complex plane \mathbb{C} and its basic geometric and topological aspects, continuity, differentiability of complex valued functions, Cauchy-Riemann (C-R) equations, analytic functions. Power series, radius of convergence and Cauchy-Hadamard theorem, infinite differentiability of sum function of power series, introduction of $\exp(z)$, $\cos z$, $\sin z$, $\log z$ and its branch-their elementary properties.

Module 5**No. of Classes: 15**

Extended complex plane \mathbb{C}_∞ , stereographic projection and spherical representation of \mathbb{C}_∞ . Bilinear transformations: The group of Mobius transformation and its generators-the inversion, dilations; fixed point and uniqueness of a Mobius transformation by its action at three distinct points; cross ratio, cross ratio and circle preserving property of Mobius transformation; orientation principle and construction of bijective analytic functions from one side of a circle onto one side of another circle in \mathbb{C}_∞ .

Group-C Metric Spaces (30 Marks)**Module 6****No. of Classes: 10**

Definition of Metric spaces, examples including the standard ones such as discrete metric space, the real line \mathbb{R} , the complex plane \mathbb{C} , Euclidian spaces \mathbb{R}^n , unitary spaces \mathbb{C}^n , (with sup metric and integral metric), .

Module 7**No. of Classes: 10**

Open ball, closed ball, metric topology, distance between a point and a set, distance between two sets, boundedness of a set, properties of open and closed sets, limit point, interior point, closure, interior, boundary of subsets and relation between them; dense subsets, nowhere dense subsets, basis, separable space, Lindelöf space, second countable space and relation between them; Hausdorff property,

Module 8**No. of Classes: 10**

Cauchy sequence, Convergence of sequences, completeness and Cantor Intersection theorem. Continuous functions and their basic properties, algebra of real/ complex valued continuous functions, uniformly continuous functions, and uniform continuity of the distance function. Open covering and compactness, compactness and finite intersection property (FIP) of closed sets, closed subsets and compactness, continuity and compactness; relation between continuity, compactness and uniform continuity.

Module 9**No. of Classes: 10**

equivalence of compactness, sequential and B-W compactness; boundedness, total boundedness and relation between them; relation between total boundedness, completeness and compactness; distance between disjoint closed sets one of which being compact, Heine Borel theorem concerning compact sets in \mathbb{R}^n .

Summary:**Total No. of Lectures: 130****Number of Tutorials: 10****No. of Class Tests: 04**

Paper –VI
Group – A Elements of Continuum Mechanics (10 marks)

Module 1

No. of Classes: 05

Idea of continuum, idea of strain and stress at a point in a continuum, stress vector, stress matrix, ideal fluid, viscous fluid.

Group – B Classical Dynamics, Dynamics of a system of Particles and rigid body (40 Marks)

Module 2

No. of Classes: 15

Physical foundation of classical dynamics: Interpretation of Newton's laws of motion – body force and surface force with examples, inertial frames, law of superposition, closed systems, concepts of absolute time, concepts of absolute space, concepts of absolute simultaneity of events; Galilean transformation – form invariance of Newton's laws under Galilean transformation, limitations of direct applications of Newton's laws in solving problems of mechanics.

Module 3

No. of Classes: 10

Dynamics of a system of particles: Basic concepts, Centroid, linear momentum, angular momentum, kinetic energy, potential energy, work, power, conservative system of forces; Use of centroid – motion relative to the centroid, angular momentum and kinetic energy relative to the centroid; Conservation principles – linear momentum, angular momentum, total energy; Constraints – basic concepts with examples, D'Alembert Principle.

Module 4

No. of Classes: 20

Introduction to rigid body dynamics: Moments and product of inertia – basic concepts, radius of gyration, parallel and perpendicular axis theorems, a few examples (rod, rectangular plate, circular plate, elliptic plate, sphere, cone, rectangular parallelepiped, cylinder, ellipsoid of revolution etc.); Motion about a point and about fixed axes – angular momentum, inertia matrix, principal axes, principal moments of inertia, kinetic energy, momental ellipsoid, equimomental surface, reaction of the axis of rotation, impulsive forces; General motion of rigid body – translational and rotational motion, kinetic energy and angular momentum (translational and rotational); Two-dimensional motion of rigid body - equation of motion, kinetic energy, angular momentum, problems illustrating laws of motion [motion of a uniform sphere (solid and hollow) along a perfectly rough plane, motion of a uniform heavy circular cylinder (solid and hollow) along a perfectly rough inclined plane, motion of a rod when released from a vertical position with one end resting upon a perfectly rough table or smooth table, motion of a uniform heavy solid sphere along an imperfectly rough inclined plane, motion of a uniform circular disc, projected with its plane vertical along an imperfectly rough horizontal plane with a velocity of translation and angular velocity about the centre

Group – C Statics (20 Marks)

Module 5

No. of Classes: 25

Forces in three-dimension – reduction to force and couple, Pointot's central axis, wrench, pitch, screw, conditions of equilibrium, invariants; Virtual work – concept of virtual displacement, principle of virtual work, simple examples; Stability of equilibrium – stable and unstable equilibrium, energy test of stability, determination of positions of equilibrium, stability of a heavy body resting on a fixed body with smooth surfaces, simple examples; Equilibrium of flexible string – general equations of equilibrium of a uniform flexible string under the action of given coplanar forces, common catenary, parabolic chain, suspension bridge, catenary of uniform strength.

Group – D Hydrostatics (30 Marks)

Module 6

No. of Classes: 35

Basic concepts – fluid pressure and its elementary properties (such as in equilibrium it is same in every direction), density, specific gravity, compressible and incompressible fluid, homogeneous and non-homogeneous fluid; Equilibrium of fluid in a given field of force – equation of pressure, conditions of equilibrium, pressure gradient, equipressure surface, equilibrium of fluid rotating uniformly about an axis; Pressure in a heavy homogeneous liquid – thrust on a plane surface, centre of pressure, determining the position of the centre of pressure, effects on increasing depth, thrust on a curved surface, buoyancy, Archimedes principle, resultant thrust, Equilibrium of floating bodies – conditions of equilibrium of a freely floating body, body floating under constraints, equilibrium of fluids revolving uniformly about an axis, stability of equilibrium, metacentre, conditions of stability;

Module 7

No. of Classes: 15

Gases – relation among pressure, volume and temperature, Boyle's law, Charle's law, ideal gas, isothermal and adiabatic changes, heat capacities, internal energy of a gas, reversible change, equilibrium of an isothermal atmosphere, convective equilibrium, total energy at rest.

Summary:

Total No. of Lectures: 125

Number of Tutorials: 10

No. of Class Tests: 04

Paper-VII

Group-A Mathematical Probability (40 Marks)

Module 1

No. of Classes: 30

Generalised addition and multiplication rule of probability continuity theory, Boole's inequality, Bonferroni's inequality; Poisson trials and Poisson law of probability, Multinomial law; Random variables, Discrete and continuous distribution functions: Poisson, Geometric, Negative Binomial, exponential, Hypergeometric, Uniform, Normal, Gamma, Beta, Cauchy distributions, Transformation of random variables; Discrete and continuous distribution in two dimension, Marginal distribution, Bivariate Uniform distribution, Bivariate Normal distribution, Transformation of two dimensional random variables, Conditional distribution,

Module 2

No. of Classes: 20

Mathematical expectation in one and two variables, Moments, Measures of skewness and kurtosis, Moment generating function, Characteristic function, Uniqueness of characteristic function (statement only) Conditional expectation, covariance, co-relation coefficient, Regression curves, χ^2 and t -distributions, convergence in probability, convergence in law, Tchebycheff's inequality, Bernoulli's limit theorem, Law of large numbers, Concept of asymptotically normal distribution, De-Moivre-Laplace limit theorem, Central limit theorem in case of equal components.

Group -B Statistics (20 Marks)

Module 3

No. of Classes: 10

Method of least square, curve fitting (straight line, parabola and exponential curves). Sampling theory, simple random sampling, sampling distribution of the statistic; χ^2 and t - and F -distribution of the statistic.

Module 4

No. of Classes: 20

Theory of estimation, point estimation, unbiasedness, minimum variance, consistency, efficiency, sufficiently, maximum likelihood method; Interval estimation –confidence interval, approximate confidence interval. Testing of hypothesis, Neyman-Pearson lemma, Likelihood ratio testing, application to Normal(m, σ)-population, Pearsonian χ^2 -test for goodness of fit. Theory of errors.

Group – C Operations Research (Marks - 40)

Module 5

No. of Classes: 05

Reduction of a feasible solution to basic feasible solution. Hyperplanes, Convex sets and their properties, Convex functions, Extreme points, Convex feasible region, Convex polyhedron, Polytope, Supporting hyperplane, Separating hyperplane.

Module 6

No. of Classes: 20

Fundamental theorem of L.P.P., Replacement of a basis vector, Improved basic feasible solutions, Unbounded solution, Condition of optimality, Simplex method, Simplex algorithm, Artificial variable technique (Big M method, Two phase method), Inversion of a matrix by Simplex method, Solution of simultaneous equations by Simplex method. Duality in L.P.P.: Concept of duality, Fundamental properties of duality, Fundamental theorem of duality, Duality & Simplex method, Dual simplex method and algorithm.

Module 7

No. of Classes: 10

Transportation Problem (T.P.): Mathematical formulation, Existence of feasible solution, Loops and their properties, Initial basic feasible solutions (different methods, like North West corner, Row minima, Column minima, Matrix minima & Vogel's Approximation method), Optimal solutions, Degeneracy in T.P., Unbalanced T.P., Special cases in T.P.

Module 8

No. of Classes: 05

Assignment Problem (A.P.): Mathematical formulation, Solution methods of A.P., Hungarian method, Restrictions on assignments, maximization problem, unbalanced assignment problem, Traveling salesman (salesperson) problem.

Module 9

No. of Classes: 10

Theory of Games: Introduction, Two person zero-sum games, Minimax and Maximin principles, Minimax and Saddle point theorems, Pure and Mixed Strategies games without saddle points, Minimax (Maximin) criterion, Dominance rules, Solution methods of games without saddle point : Algebraic method, Graphical method and Linear Programming method, Symmetric game.

Summary:

Total No. of Lectures: 130

Number of Tutorials: 10

No. of Class Tests: 04

Paper – VIII

Group A Numerical Analysis (35 Marks)

Module 1

No. of Classes: 05

Approximation of numbers, significant digits, Loss of significance, Algebraic manipulation for avoiding loss of significance. Errors: Absolute, Relative and Percentage errors; Inherent errors in numerical methods.

Module 2

No. of Classes: 10

Polynomial Interpolations: Existence and uniqueness of interpolating polynomials, error in interpolation, Lagrange's interpolating formula, Newton's divided difference interpolating formula, properties of divided differences, forward and backward difference operators and their relations, Newton's forward and backward difference interpolation formulae. Central difference and averaging operators, central interpolation formulae: Statement of Gauss, Stirling and Bessel's formulae and their applications. Concept of piece-wise polynomial interpolation, Idea of Inverse interpolation.

Module 3

No. of Classes: 10

Numerical solution of non-linear equations: Solution of algebraic and transcendental equations (real roots only): (i) Method of Bisection, (ii) Regula Falsi Method (iii) Secant Method (iv) Newton – Raphson Method (v) Fixed point iteration method. Convergences and rate of convergence of these methods.

Module 4

No. of Classes: 10

Solution of a system of linear algebraic equation: Gauss' Elimination and Gauss Jordan methods, Pivoting methods, Jacobi and Gauss-Seidel methods with convergence criteria. Numerical Integration: Concept of numerical quadrature, Newton-Cotes' formula trapezoidal rule, Simpson's one-third rule, Geometrical interpretation of the methods, Degree of precision.

Module 5

No. of Classes: 10

Solution of first-order ordinary differential equation: Picard's method, Euler's method, Modified Euler's method, Error estimate and convergence of Euler's method, Taylor's method, Runge-Kutta's method of second and fourth orders (derivation of second order formula only)

Group-B Computer Programming (Marks – 15)

Module 6

No. of Classes: 20

Computer Language: Concept of programming languages, Machine language, Assembly language, High-level language, Interpreter, Compiler, Source and Object programs. Number Systems: Binary,

decimal, octal and hexadecimal number systems and their conversions. Programming Language in C: C-Character set, Keywords, Basic data types, Numeric constants and variables operators, Expressions, Assignment statements, I/O – statements. Control Statements: Decision making and Looping statements in C, break continue and goto statements, Example of simple programs. Subscripted variables: Concept of array variables in programming language, Rules for one dimensional subscripted variable in C, Simple programs. Sub-program: Concept of sub-program, purpose of sub-program, Definition of function and function prototype, Simple programs.

Summary:

Total No. of Lectures: 165

Number of Tutorials: 05

No. of Class Tests: 02

Paper - IX
(Computer Aided Numerical Methods -Practical)
Computer Aided Numerical Methods - Practical (Marks- 50)

Module 1

No. of Practical: 04

Prerequisites : PC – operating system and DOS commands, Concepts of Algorithms, Flowchart and Subscripted variables

Module 2

No. of Practical: 04

1. Finding a real Root of an equation by
 - (a) Fixed point iteration and
 - (b) Newton-Rapson's method.
2. Finding the solution of linear equations by Gauss-Seidel method
3. Interpolation (Taking at least six points) by Lagrange's formula

Module 3

No. of Practical: 04

4. Integration by
 - (i) Trapezoidal rule
 - (ii) Simpson's $1/3^{\text{rd}}$ rule (taking at least 10 sub-intervals)
5. Solution of a 1^{st} order ordinary differential equation by fourth-order R. K. Method, taking at least four steps.

Summary:

Total No. of Practical: 12

No. of Class Tests: 02

DEPARTMENT OF MATHEMATICS
COURSE MODULE
FOR
MATHEMATICS (HONOURS) COURSE

Old Syllabus

Paper – I
(Abstract Algebra, Linear Algebra and Number Theory)

Group – A

Abstract Algebra (Marks - 50)

Module 1

No of Lectures 20

Sets, relation, mappings – surjective, injective and bijective, Composition of two mappings, extension and restriction of a mapping ; Countable and uncountable sets, countability of rational numbers and uncountability of the reals. Equivalence relation and partition of a set, partial order relation. Maximal and minimal elements, infimum and supremum of subsets, uniqueness, Hasse's diagram, Lattices as p.o. set, definition of lattice in terms of meet and join, equivalence of two definitions, linear order relation

Module 2

No of Lectures 05

Boolean algebra and its application to simple circuit.

Module 3

No of Lectures 30

Groups – definition, Dihedral group subgroups, generators of groups and subgroups, order of a group and order of an element, Abelian groups.
Permutation groups, cycles, length of a cycle, transpositions, even and odd permutation, alternating group, important examples such as S_3 and K_4 (Klein4 –group),
Cyclic subgroup of a group, cyclic groups and their subgroups, Cyclic subgroups of prime order, Cayley's theorem.
Cosets, normal subgroup, quotient group, Lagrange's theorem, homomorphism, isomorphism, kernel of homomorphism, First homomorphism theorem, isomorphism of cyclic groups.

Module 4

No of Lectures 30

Ring, subring, ideal of a ring, ring homomorphism elementary properties, Integral domain, Characteristic of a ring
Field, subfield, finite field, characteristics of a field, Every integral domain can be extended to a field.

Group B

Linear Algebra (Marks - 30)

Module 5

No of Lectures 6

Matrices, addition and multiplication of matrices, row equivalence, row reduced echelon form, elementary matrices, nonsingular matrices, inverse, Hermitian, Skew Hermitian, orthogonal matrices.

Module 6**No of Lectures 18**

Vector space, linearly dependent and independent vectors, subspaces, span of a subset, basis, dimensions of a finite dimensional vector space, change of coordinates. Row rank and column rank of a matrix, rank of a matrix. Determinants and its properties, Laplaces' expansions of determinants (proof not required), product of determinants (proof not required).

Linear transformation, algebra of linear transformation. existence of solution of homogeneous and non-homogeneous system of linear equations and determination of their solution.

Module 7**No of Lectures 11**

Characteristic equation, statement of Caley-Hamilton theorem and its application like inverse and powers of a non-singular matrices, eigen values, eigen vectors, similar matrices, similarity transformation, diagonalization of matrices of order 2 and 3 with application to Geometry.

Group C**Number Theory (Marls 20)****Module 8****No of Lectures 08**

Primes and composite numbers, Fundamental theorem of arithmetic, greatest common divisor, relatively prime numbers, Euclid's algorithm, least common multiple.

Module 9**No of Lectures 08**

Congruences : properties and algebra of congruences, power of congruence, Fermat's congruence, Fermat's theorem, Wilson's theorem, Euler – Fermat's theorem, Chinese remainder theorem.

Module 10**No of Lectures 09**

Number of divisors of a number and their sum, least number with given number of divisors. Eulers ϕ function - $\phi(n)$. Mobius μ - function, relation between ϕ function and μ function. Diophantine equations of the form $ax+by = c$, a, b, c integers.

Summary:**Total No. of Lectures: 145****Number of Tutorials: 10****No. of Class Tests: 05**

Paper – II
(Analysis)

Group – A

Differential Calculus (Marks - 50)

Module 1

No of Lectures 10

Order completeness of real numbers system is to be assumed. Sequence of real numbers, Monotone Sequence, Notion of convergence, Upper and Lower Limits of a Sequence, Limits, Algebra of Limits. Subsequences and their convergence.

Module 2

No of Lectures 10

Series of nonnegative terms, Test for convergence, Ratio test, Comparison test, Cauchy's root test, Raabe's test, Gauss's test, Leibnitz test. Series of arbitrary numerical terms, Alternating series, Absolutely and conditionally Convergent series, Riemann's rearrangement theorem (Proof not required).

Module 3

No of Lectures 15

ϵ - δ definition of limit of a function at a point, Algebra of limits, Bounded function, Monotone function, Continuous function, Local continuity, Properties of continuous function over a closed interval (without proof), Derivative, Successive differentiation, Leibnitz's theorem.

Module 4

No of Lectures 10

Rolle's theorem, Mean value theorems, Intermediate value property of the derivatives, Taylor and Maclaurin's theorem with Cauchy and Lagrange's form of remainder, Taylor series, Expansion of elementary function such as e^x , $\cos x$, $\sin x$, $(1+x)^n$, $\log_e(1+x)$ etc.

Module 5

No of Lectures 10

Tangent, Normal, Envelope, Asymptote, Curvature, Curve tracing; Astroid, Cycloid, Cardinal Folium of Descartes.

Maxima, Minima, Indeterminate form, L'Hospital's theorem.

Module 6

No of Lectures 05

Functions of Several variables, Continuity, differentiability, Partial derivative, Commutativity of the orders of partial derivatives [Schwartz's theorem only], Euler's theorem.

Group B**Integral Calculus (Full marks 20)****Module 7****No of Lectures 10**

Indefinite integrals and statement of their properties, Method of partial fraction, Reduction formula. Statement of properties of definite integral and its applications.

Module 8**No of Lectures 15**

Double and triple integration, Rectification, Quadrature. Pappus theorem (proof not required). Calculation of volumes and surfaces of revolution, Evaluation of integrals for center of gravity of symmetric configuration including an arc, sector of a circle and hemisphere.

Group – C**Vector Analysis (30 Marks)****Module 9****No of Lectures 20**

Vector Algebra ;Addition of vectors, Scalar and Vector products of two vectors, Scalar and vector triple products, Geometrical interpretations of different Products. Representation of a vector in E_3 , Components and resolved parts of vectors. Point of division of a line segment, Vector equation of a straight line & a plane.

Module 10**No of Lectures 20**

Continuity and differentiability of vector-valued function of one variable. Velocity and acceleration. Space curve, Arc length, Tangent, Normal. Integration of vector-valued function of one variable. Vector-valued functions of two and three variables, Gradient of scalar function, Gradient vector as normal to a surface. Divergence and curl, their properties. Evaluation of line integral of different types.

Module 11**No of Lectures 20**

Green's theorem in the plane. Gauss and Stokes, theorems (Proof not required), Green's first and second identities. Evaluation of surface integrals of different types.

Summary:**Total No. of Lectures: 145****Number of Tutorials: 10****No. of Class Tests: 05**

Part – II

Paper - III

(Classical Algebra, Geometry and Tensor Calculus)

Group – A

Classical Algebra (Marks - 30)

Module 1

No. of Lectures: 15

Polynomial equation, Fundamental theorem of Algebra (Statement only), Multiple roots, Statement of Rolle's theorem only and its applications, Equation with real coefficients, Complex roots, Descarte's rule of sign, relation between roots and coefficients, transformation of equation, reciprocal equation, binomial equation – special roots of unity, solution of cubic equations – Cardan's method, solution of biquadratic equation – Ferrari's method.

Module 2

No. of Lectures: 15

Inequalities involving arithmetic, geometric and harmonic means, Schwarz and Weierstrass's inequalities.
Simple Continued fraction and its convergents, representation of real numbers.

Module 3

No. of Lectures: 10

Complex numbers, De Moivre's theorem. Direct and inverse circular and hyperbolic functions, logarithm of a complex quantity.

Group – B

Geometry (Marks-50)

Two Dimensional Geometry: 20

Module 4

No. of Lectures: 20

Transformation of Co-ordinates, Invariants, General equation of second degree, Pair of St. lines, Classification of Conics, Pole, Polar, Polar Co-ordinates, Polar equation of st. lines, Circles, Conics, tangents, normals to Conics and their properties.

Three Dimensional Geometry: 30

Module 5

No. of Lectures: 25

Rectangular Cartesian Co-ordinates, Transformation of Co-ordinates, Invariants, Planes, St. lines, Shortest distance between two lines, Spheres, Cones, Cylinders, Ellipsoid, Paraboloid, hyperboloid. Tangent Planes and Normals and their properties, Central Quadrics, Ruled Surfaces, Generators, their properties, Classification of Quadrics.

Group – C Tensor Calculus (Marks - 20)

Module 6

No. of Lectures: 15

Tensor as a generalized concept of a vector in E_3 . Generalization of idea to an n-dimensional Euclidean space (E_n), Definition of an n-dimensional space, Transformation of Co-ordinates, Summation Convention, Kronecher delta, Invariant, Contravariant and Covariant vectors, symmetric tensors, Contraction, Outer and inner products of tensors, Quotient law (Statement only). Fundamental metric tensor of Riemannian space, Reciprocal metric tensor. A magnitude of a vector, angle between two vectors, Christoffel symbols, Covariant differentiation of vectors and tensors of rank 1 and 2. The identities $g_{i,j,k}^{ij} = g_{,k}^{ij} = 0$ and $S_{j,k}^i = 0$

Summary:

Total No. of Lectures: 100

Number of Tutorials: 12

No. of Class Tests: 05

Paper – IV
(Differential Equation and Dynamics of a Particle)

Group – A Differential Equation (Marks - 50)

Ordinary Differential Equations: (Marks - 40)

Module 1

No. of Lectures: 10

Definition of ordinary differential equation-its order and degree, Formation of ordinary differential equations by elimination, Examples of differential equation from various fields of Sciences. First order linear equation, I.F., Exact differential Equations, condition of integrability. Solution of differential equation. Complete primitive and particular integral of ordinary differential equation.

Module 2

No. of Lectures: 15

Linear differential equation-Linear property of its solutions. Picard's existence theorem (statement only) for $\frac{dx}{dy} = f(x,y)$ with $y = y_0$ at $x = x_0$. Equation of first order and first degree-exact equations and those convertible to exact form. Solution by separation of variables. Homogeneous equations. Linear equations of first order. Equations of first order but not of first degree-equations solvable for $p = \frac{dx}{dy}$, equations solvable for y , equation solvable for x , singular solutions, Clairart's form. Singular solution as envelope to family of general solution to the equation.

Module 3

No. of Lectures: 15

Linear differential equation of second and higher order. Two linearly independent solutions of a second order linear differential equations and Wronskian, general solution of second order linear differential equation, solution of linear differential equation of second order with constant coefficients. Solution when values of y and x are given at a point. Particular integrals for second order linear differential equation with constant coefficients for polynomial, sine, cosine, exponential function and for function as combination of them or involving them. Method of variation of parameters for P.I. of linear differential equation of second order. Homogeneous linear equation of n -th order with constant coefficients. Reduction of order of linear differential equation of second order when one solution is known.

Module 4

No. of Lectures: 10

Simultaneous linear ordinary differential equation in two dependent variables. Solution of simultaneous equations of the form $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$. Equation of the form (Paffian form) $Pdx + Qdy + Rdz = 0$. Necessary and sufficient condition for existence of integrals of the above. Total differential equation.

Partial Differential Equations : (Marks - 10)

Module 5

No. of Lectures: 15

Their formations, Lagrange's Linear equation. General integral and complete integral. Integral surface passing through a given curve.

Group – B Dynamics of a Particle (Marks- 50)

Module 6

No. of Lectures: 10

Kinematics

Expressions for velocity & acceleration for

- (i) Motion in a straight line;
- (ii) Motion in a plane;
- (iii) Motion in three dimension in rectangular cartesian co-ordinates.

Expressions for velocity & acceleration for motion in a plane

- (i) referred to rotating axes in the plane,
- (ii) in plane polar co-ordinates,
- (iii) in tangential and normal direction of the path of the particle.

Module 7

No. of Lectures: 20

Kinetics

Momentum and Angular momentum of a moving particle, Newton's laws of motion. Equation of motion of a particle moving under the action of given external forces. Forces of friction. Rolling and Sliding friction. Nature of force in light inextensible string and in a light elastic string. Work and Power. Conservative force field. Potential energy and Kinetic energy of a particle. Principles of conservation

- (i) of linear momentum, (ii) of angular momentum, (iii) of energy of a particle

Impulse of force. Impulsive forces, change of momentum under impulsive forces. Examples. Collision of two smooth elastic bodies. Newton's experimental law of impact. Direct and oblique impacts of (i) Sphere on a fixed horizontal plane, (ii) Two smooth spheres, Energy loss.

Motion of a particle in a straight line in the following cases :

For forces of the forms

- (a) μx^n , $n = 0, \pm 1, n = -2$ ($\mu > 0$ or < 0) with physical interpretation.
- (b) S.H.M. of a particle attached to one end of an elastic string, the other end being fixed.
- (c) Linearly damped harmonic motion.
- (d) Forced oscillation with and without damping.
- (e) Vertical motion under gravity when resistance varies as the velocity or the square of the velocity.
- (f) Motion with constant power under resistance proportional to velocity under gravity.
- (g) Vertical motion of a particle when its mass changes at a constant rate
- (h) Motion of a heavy particle along a smooth or rough inclined plane.

Motion in two dimensions:

Module 8

No. of Lectures: 20

- (i) Motion of a projectile under gravity with air resistance neglected ;
- (ii) Motion of a projectile under gravity with air resistance proportional to velocity, square of the velocity ;
- (iii) Motion of a simple pendulum ;

(iv) Central Orbit – Motion under a central force: basic properties and differential equation of the path under given forces and velocity of projection. Apses. Time to describe a given arc of an orbit.

Law of force when the center of force and the central orbit are known. Special study of the following problems :

To find the central force for the following orbits –

- (a) A central conic with the force directed towards the focus ;
- (b) Equiangular spiral under a force to the pole ;
- (c) Circular orbit under a force towards a point on the circumference.

To determine the nature of the orbit and of motion for different velocity of projection under a force per unit mass equal to –

- (a) $\mu / (\text{dist})^2$ towards a fixed point ;
- (b) under a repulsive force $\mu / (\text{dist})^2$ away from a fixed point

Module 9

No. of Lectures: 15

Circular orbit under any law of force $\mu f(r)$ with the centre of the circle as the centre of force.

Question of stability of a circular orbit under a force $\mu f(r)$ towards the center. Particular case of $\mu f(r) = 1/r^n$, Kepler's laws of planetary motion from the equation of motion of a central orbit under inverse square law. Modification of Kepler's third law from consideration of motion of a system of two particles under mutual attractions according to Newton's law of gravitational attraction. Escape velocity.

Motion of a particle along a smooth curve. Examples of motion under gravity along a smooth vertical circular curve, smooth vertical cycloidal arc (cycloidal pendulum), parabolic curve. Motion of a particle along a rough curve (circle, cycloid) & in a resisting medium.

Summary:

Total No. of Lectures: 130

Number of Tutorials: 14

No. of Class Tests: 04

Part – III
Paper – V
(Analysis – II)

Group – A Metric Spaces (Marks - 40)

Module 1

No. of Lectures: 10

Metric, examples of standard metric spaces including Euclidean and Discrete metrics; open ball, closed ball, open sets; metric topology; closed sets, limit points, and their fundamental properties; interior, closure and boundary of subsets and their interrelation; denseness; separable and second countable metric spaces and their relationship.

Module 2

No. of Lectures: 15

Continuity : Definition of continuous functions, algebra of real/complex valued continuous functions, distance between a point and a subset, distance between two subsets.

Connectedness: Connected subsets of the real line \mathbb{R} , open connected subsets in \mathbb{R}^2 , components; components of open sets in \mathbb{R} and \mathbb{R}^2 ; Structure of open set in \mathbb{R} , continuity and connectedness; intermediate value theorem.

Sequence and completeness: Sequence, subsequence and their convergence; Cauchy sequence and completeness, completeness of \mathbb{R}^n ; Cantor's theorem concerning completeness. Definition of completion of a metric space, construction of the reals as the completion of the incomplete metric space of the rationals with usual distance (proof not required). Continuity preserves convergence.

Module 3

No. of Lectures: 10

Compactness: Definitions (by means of open covering), Compact metric spaces and finite intersection property (FIP) of closed sets; Compact subsets, continuity and compactness; sequential compactness, Equivalence between compactness and sequential compactness, relation between compactness, completeness and total boundedness. Heine-Borel theorem concerning characterization of compact subsets of \mathbb{R}^n . Uniform continuity and continuity on compact sets; distance between two non empty disjoint closed set one of which is compact is a positive real.

Group – B Real Analysis (Marks - 40)

Module 4

No. of Lectures: 15

Definition of Riemann integration, Uniqueness, Cauchy's criterion, Linear property, Darboux theory of Riemann integration, equivalence, Darboux theorem (proof not required), Riemann integral as the limit of a sum, equivalence. Fundamental theorem of integral calculus, Properties of the Riemann integral; Riemann integrability of continuous and monotone functions, discontinuous function.

Module 5**No. of Lectures: 15**

First and second Mean value theorems of Integral Calculus. Functions defined by integrals, their continuity and differentiability.

Convergence of sequence and series of functions, uniform convergence, Cauchy's Criterion of uniform convergence, continuity of sum function of a uniformly convergent series of continuous functions, term by term differentiation and integration for proper integrals.

Module 6**No. of Lectures: 15**

Functions of several variables, theory of extrema, maxima, minima, Lagranges' method of multipliers, Jacobian, Implicit function theorem (proof not required).

Integral as a function of parameter. Differentiation and integration under the sign of integration, change of order of integration for repeated integrals.

Improper integrals, their convergence (for unbounded functions and unbounded range of integration) Abel's and Dirchlets' test, Beta and Gamma function, Evaluation of improper integrals

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2}, \int_0^{\infty} \frac{x^{\alpha-1} dx}{1+x^{\alpha}}, \int_0^{\infty} \frac{x^{\alpha-1} dx}{e^x}, \alpha > 0$$

and integrals dependent on them.

Module 7**No. of Lectures: 10**

Fourier series associated with a function, Series of odd and even functions, Main theorem concerning Fourier series expansion of piece wise monotone functions (proof not required).

Group – C Complex Analysis (Marks - 20)

Module 8**No. of Lectures: 15**

Introduction of complex number as ordered pair of reals, geometric interpretation, metric structure of the complex plane \mathbb{C} , regions in \mathbb{C} . Stereographic projection and extended complex plane \mathbb{C}_{∞} and circles in \mathbb{C}_{∞}

Continuity and differentiability of a complex function. Analytic functions and Cauchy Riemann equation, harmonic functions.

Module 9**No. of Lectures: 10**

Power series, radius of convergence, sum function and its analytic behaviour within the circle of convergence, Cauchy-Hadamard Theorem.

Introduction of $\exp(z)$, $\sin z$, $\cos z$, $\tan z$ and the branches of $\log z$ and their analytic behaviour.

Transformation (mapping), Concept of Conformal mapping, Bilinear (Möbius) transformation and its geometrical meaning, fixed points and circle preserving character of Möbius transformation.

Summary:

Total No. of Lectures: 115

Number of Tutorials: 10

No. of Class Tests: 04

Paper – VI
(Mechanics)

Group – A

Principles of Mechanics (Marks - 70)

Module 1

No. of Lectures: 10

I. Physical Foundations of Classical Dynamics: (Marks - 10)

Inertial frames, Newton's laws of motion, Galilean transformation. Form-invariance of Newton's laws of motion under Galilean transformation. Fundamental forces in classical physics (gravitation). Electric and Magnetic forces, action-at-a-distance. Body forces; contact forces: Friction, Viscosity.

Module 2

No. of Lectures: 25

II. Dynamics of a system of particles and of a rigid body (Vector treatment): (Marks - 40)

System of particles:

Fundamental concepts, centre of mass, momentum, angular momentum, kinetic energy, work done by a field of force, conservative system of forces – potential and potential energy, internal potential energy, total energy.

Following important results to be deduced :

- (i) Centre of mass moves as if the total external force were acting on the entire mass of the system concentrated at the centre of mass (examples of exploding shell, jet and rocket propulsion).
- (ii) The total angular momentum of the system about a point is the angular momentum of the system concentrated at the centre of mass, plus the angular momentum for motion about the center.
- (iii) Similar theorem as in (ii) for kinetic energy.

Conservation laws : conservation of linear momentum, angular momentum and total energy for conservative system of forces.

An idea of constraints that may limit the motion of the system, definition of rigid bodies.

D'Alembert's principle, principle of virtual work for equilibrium of a connected system.

Module 3

No. of Lectures: 20

Rigid Body :

Moments and products of inertia (in three-dimensional rectangular co-ordinates). Inertia matrix. Principal values and principal axes of inertia matrix. Principal moments and principal axes of inertia for (i) a rod, (ii) a rectangular plate, (iii) a circular plate, (iv) an elliptic plate, (v) a sphere, (vi) a right circular cone, (vii) a rectangular parallelepiped and (viii) a circular cylinder.

Two-dimensional motion of a rigid body. Following examples of the two-dimensional motion of a rigid body to be studied :

- (i) Motion of a uniform heavy sphere (solid and hollow) along a perfectly rough inclined plane;
- (ii) Motion of a uniform heavy circular cylinder (solid and hollow) along a perfectly rough inclined plane:

- (iii) Motion of a rod when released from a vertical position with one end resting upon a perfectly rough table or smooth table.
- (iv) Motion of a uniform heavy solid sphere along an imperfectly rough inclined plane ;
- (v) Motion of a uniform circular disc, projected with its plane vertical along an imperfectly rough horizontal plane with a velocity of translation and angular velocity about the centre.

Module 4

No. of Lectures: 15

III. Analytical Statics : (Marks -20)

Forces, concurrent forces, Parallel forces. Moment of a force, Couple. Resultant of a force and a couple (Fundamental concept only).

Reduction of forces in three-dimensions, Pointsof's central axis, conditions of equilibrium. Virtual work, Principle of Virtual work.

Simple examples of finding tension or thrust in a two-dimensional structure in equilibrium by the principle of virtual work.

Stable and unstable equilibrium- Energy test of stability, stability of a heavy body resting on a fixed body with smooth surfaces- simple examples.

General equations of equilibrium of a uniform heavy inextensible string under the action of given coplanar forces, common catenary, catenary of uniform strength.

Module 5

No. of Lectures: 10

Group – B Elements of Continuum Mechanics with Hydrostatics (Marks - 30)

1. Elements of Continuum Mechanics:

Deformable body. Idea of a continuum (continuous medium). Surface forces or contact forces. Stress at point in a continuous medium, stress vector, components of stress (normal stress and shear stress) in rectangular Cartesian co-ordinate system; stress matrix. Definition of ideal fluid and viscous fluid.

Module 6

No. of Lectures: 20

II. Hydrostatics :

Pressure (pressure at a point in a fluid in equilibrium is same in every direction). Incompressible and compressible fluid, Homogeneous and non-homogeneous fluids.

Equilibrium of fluids in a given field of force; pressure gradient. Equipressure surfaces, equilibrium of a mass of liquid rotating uniformly like a rigid body about an axis. Simple applications.

Pressure in a heavy homogeneous liquid. Thrust on plane surfaces: center of pressure, effect of increasing the depth without rotation. Centre of pressure of a triangular & rectangular area and of a circular area immersed in any manner in a heavy homogeneous liquid. Simple problems.

Thrust on curved surfaces :Archemedes' principle. Equilibrium of freely floating bodies under constraints. (Consideration of stability not required).

Equation of state of a 'perfect gas', Isothermal and adiabatic processes in an isothermal atmosphere.

Pressure and temperature in atmosphere in convective equilibrium.

Summary:

Total No. of Lectures: 100

Number of Tutorials: 10

No. of Class Tests: 04

Fundamental theorems of L.P.P., Replacement of a basis vector, Improved basic feasible solutions, Unbounded solution, Condition of optimality, Simplex method, Simplex algorithm, Artificial variable technique (Big M method, Two phase method), Inversion of a matrix by Simplex method. Duality in L.P.P. : Concept of duality, Fundamental properties of duality, Fundamental theorem of duality, Duality & Simplex method, Dual simplex method and algorithm.

Module 5

No. of Lectures: 15

Transportation Problem (T.P.) : Matrix form of T.P., the transportation table, Initial basic feasible solutions (different methods like North West corner, Row minima, Column minima, Matrix minima & Vogel's Approximation method), Loops in T.P. table and their properties, Optimal solutions, Degeneracy in T.P., Unbalanced T.P.

Module 6

No. of Lectures: 20

Theory of Games : Introduction, Two person zero-sum games, Minimax and Maximin principles, Minimax and Saddle point theorems, Mixed Strategies games without saddle points, Minimax (Maximin) criterion, The rules of Dominance. Solution methods of games without Saddle point : Algebraic method, Matrix method, Graphical method and Linear Programming method.

Summary:

Total No. of Lectures: 80

Number of Tutorials: 10

No. of Class Tests: 04

Paper – VIII
(Numerical Analysis and Computer Programming)
Group – A Numerical Analysis (Marks - 35)

Module 1

No. of Lectures: 10

Approximation of numbers, decimal places, significant figures. Round off. errors in numerical calculations. addition, subtraction, multiplication and division. Loss of significant figures, Inherent errors in numerical methods. Ordinary and divided differences, Propagation of error in difference table. Problems of interpolation, remainder in interpolation. Newton's forward and backward interpolation formulae. Newton's divided difference formula. Central interpolation formulae: Gauss, Stirling and Bessel's formulae (Deduction not necessary).
Lagrange's interpolation formula. Inverse interpolation formula.

Module 2

No. of Lectures: 20

Numerical integration : Newton-Cotes' formula (error term may be stated). Trapezoidal rule, Simpson's one-third rule, Inherent errors, degree of precision.
Numerical methods for finding the real roots of algebraic and transcendental equations :Location of roots by Tabulation and Graphical method. Finding the roots by the method of (i) Regula-Falsi (ii) Fixed point iteration and (iii) Newton Raphson & their convergences.
Solution of a system of linear equation: Gauss' elimination method and Gauss-Seidel method; statement of convergence criteria.

Module 3

No. of Lectures: 10

Solution of first order ordinary differential equations: Picard's method, Euler's method (modified), Taylor's method and Runge-Kutta's method of second and fourth order (derivation of 2nd order formula only).

Group – B Computer Programming (Marks – 15)

Module 4

No. of Lectures: 10

Anatomy of a computer: Basic structure, Input unit, Output unit, Memory unit, Control unit, Arithmetic logical unit. Computer generation and classification; Machine language, Assembly language, computer-high level languages. Compiler, Interpreter, Operating system.. Source programs and objects programs. Binary number system, Conversions and arithmetic operation.
Representation for Integers and Real numbers, Fixed and floating point.

Module 7

No. of Lectures: 10

Programming in FORTRAN-77 Language: Fortran Characters. Basic data types; Numeric Constant & Variables; Arithmetic Expressions, Assignment statements, I/O –statements(Format-free) ; STOP & END statement; Control statement: Unconditional GOTO, Computed GOTO, Assigned GOTO, Logical IF and Arithmetic IF.

Repetitive operations: DO statement; CONTINUE statement, Arithmetic statement functions;
Library functions in FORTRAN.

Summary:

Total No. of Lectures: 60

Number of Tutorials: 10

No. of Class Tests: 03

Paper - IX
(Computer Aided Numerical Methods -Practical)
Computer Aided Numerical Methods - Practical (Marks- 50)

Module 1

No. of Practical: 04

Prerequisites : PC – operating system and DOS commands, Concepts of Algorithms, Flowchart and Subscripted variables

Module 2

No. of Practical: 04

1. Finding a real Root of an equation by
 - (a) Fixed point iteration and
 - (b) Newton-Rapson's method.
2. Finding the solution of linear equations by Gauss-Seidel method
3. Interpolation (Taking at least six points) by Lagrange's formula

Module 3

No. of Practical: 04

4. Integration by
 - (i) Trapezoidal rule
 - (ii) Simpson's $1/3^{\text{rd}}$ rule (taking at least 10 sub-intervals)
5. Solution of a 1^{st} order ordinary differential equation by fourth-order R. K. Method, taking at least four steps.

Summary:

Total No. of Practical: 12

No. of Class Tests: 02
